Review for Chapter 8
Important Terms, Symbols, Concepts

- 8.1. Sample Spaces, Events, and Probability
  - Probability theory is concerned with **random experiments** for which different outcomes are obtained no matter how carefully the experiment is repeated under the same conditions.
  - The set *S* of all possible outcomes of a random experiment is called a **sample space**. The subsets of *S* are called **events**. An event that contains only one outcome is called a **simple event**. Events that contain more than one outcome are **compound events.**

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- 8.1. Sample Spaces, Events, and Probability (continued)
  - If $S = \{e_1, e_2, \ldots, e_n\}$ is a sample space for an experiment, an **acceptable probability assignment** is an assignment of real numbers $P(e_i)$ to simple events such that $0 \le P(e_i) \le 1$ and $P(e_1) + P(e_2) + \ldots + P(e_n) = 1$.
  - Each number $P(e_i)$ is called the **probability of the simple event** $\boldsymbol{e_i}$. The **probability of an arbitrary event** $\boldsymbol{E}$, denoted $P(E)$, is the sum of the probabilities of the simple events in E. If E is the empty set, then $P(E) = 0$.

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- 8.1. Sample Spaces, Events, and Probability (continued)
  - Acceptable probability assignments can be made using a **theoretical** approach or an **empirical** approach. If an experiment is conducted n times and event E occurs with frequency $f(E)$, then the ratio $f(E)/n$ is called the **relative frequency** of the occurrence of E in n trials, or the **approximate empirical probability of *E*.**
  - If the **equally likely assumption** is made, each simple event of the sample space $S = \{e_1, e_2, \ldots, e_n\}$ is assigned the same (theoretical) probability $1/n$.

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- 8.2. Union, Intersection, and Complement of Events; Odds
  - Let A and B be two events in a sample space S. Then $A \cup B = \{x \mid x \in A \text{ or } x \in B\}$ is the **union** of A and B; $A \cap B = \{x \mid x \in A \text{ and } x \in B\}$ is the **intersection** of A and B.
  - Events whose intersection is the empty set are said to be **mutually exclusive** or **disjoint**.
  - The probability of the union of two events is given by $P(A \cup B) = \mathrm{P}(A) + P(B) - P(A \cap B)$.

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- 8.2. Union, Intersection, and Complement of Events; Odds (continued)
  - The **complemen**t of event E, denoted E', consists of those elements of S that do not belong to E.

    $$P(E') = 1 - P(E)$$

  - The language of **odds** is sometimes used, as an alternative to the language of probability, to describe the likelihood of an event. If $P(E)$ is the probability of E, then the **odds for *E*** are $P(E)/P(E')$, and the **odds against *E*** are $P(E')/P(E)$.
  - If the odds for an event are a/b, then $P(E) = a/(a+b)$.

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- 8.3. Conditional Probability, Intersection, and Independence
  - If A and B are events in a sample space S, and $P(B) \neq 0$, then the **conditional probability of *A* given *B*** is defined by

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

  - By solving this equation for $P(A \cap B)$ we obtain the **product rule** $P(A \cap B) = P(B)\, P(A|B) = P(A)\, P(B|A)$
  - Events A and B are **independent** if $P(A \cap B) = P(A)\, P(B)$

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- 8.4. Bayes' Formula

Let $U_1, U_2, \ldots U_n$ be n mutually exclusive events whose union is the sample space S. Let E be an arbitrary event in S such that $P(E) \neq 0$. Then

$$P(U_1|E) = \frac{\text{product of branch probabilities leading to } E \text{ through } U_1}{\text{sum of all branch products leading to } E}$$

Similar results hold for $U_2, U_3, \ldots U_n$

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- 8.5. Random Variable, Probability Distribution, and Expected Value
  - A **random variable** *X* is a function that assigns a numerical value to each simple event in a sample space *S*.
  - The **probability distribution of *X*** assigns a probability $p(x)$ to each range element x of X: $p(x)$ is the sum of the probabilities of the simple events in S that are assigned the numerical value x.

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- 8.5. Random Variable, Probability Distribution, and Expected Value (continued)
  - If a random variable X has range values $x_1, x_2, \ldots, x_n$ which have probabilities $p_1, p_2, \ldots, p_n$, respectively, the **expected value of** $\boldsymbol{X}$ is defined by

$$E(X) = x_1 p_1 + x_2 p_2 + \ldots + x_n p_n$$

  - Suppose the x_i's are payoffs in a game of chance. If the game is played a large number of times, the expected value approximates the average win per game.